MATHEMATICAL METHODS OF OPERATIONS RESEARCH

Mathematical Methods of Operations Research

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To Bernadine

PREFACE

Operations research—the scientific approach to operational problems for the greater fulfillment of objectives—is a parallel development to an increased realization of the usefulness of staff planning and analysis functions. The implementation of these functions, under the name of operations research or otherwise, has led to rapid development of new methodology and application of old methodology to new problems. The books so far available in the field have concentrated mainly on case histories. The fact that operations research utilizes a great variety of mathematical methods has been an incentive for writing this book.

The practice of operations research involves more than mathematical techniques. However, problems usually require analytical treatment, which then enables one to form conclusions with greater confidence.

The material in this book was gradually developed, over a period of several years, as a set of notes for a course in operations research adapted to the widely different backgrounds and specialities of the students taking the course. Compromise in the presentation had to be made on many occasions. Some elements of this compromise remain in the book because it was felt that there would be a similar variety of background among the readers.

For those readers who are interested in operations research but lack adequate mathematical training, many problems are posed in elementary fashion in order to give, if possible, better insight and to stimulate suggestions for probable solutions.

The mathematical methods of operations research encompass most of applied mathematics and some pure mathematics, e.g., basic set theory, lattice theory, etc. To include all these methods would be outside the scope of the book.

The book requires for background a course in calculus, with some elements of advanced calculus and rudimentary knowledge of matrix theory. There are a few places in which a degree of analytical experience would be helpful.

Chapters 1 to 4 provide some background material and also serve to complete the "perspective." A chapter on some mathematical models is included to illustrate, by brief discussions and examples, how the need

for such tools arises and some types of theoretical questions associated with them which also require answers. This chapter will serve as a starting point for expanding these methods wherever desirable.

Because of the variety of methods, a greater degree of completeness has been attempted in those subjects which it was felt needed such unity as far as the literature is concerned. The Part on optimization is an illustration of this.

Although the material in the two chapters on probability and statistics is well documented in the existing literature, it seemed best to give a unified treatment of the analytical methods most commonly used in operations research and to provide some of the needed background for the chapters on applications. The brief presentations on probability and statistics are meant for the beginning student, and only as guides—not as last words. The interested reader has many references to use in expanding his knowledge of the subject. Examples which were available and illustrative have been used from the examined literature.

Although it is easier to present systematically only some of the analytical methods available, an attempt was made to include material on creative imagination. In the final analysis, the individual himself, his talents and methods of approaching problems, form the fundamental qualities required in solving problems. Those analytical methods studied give a reliable and powerful approach to problems.

However, many of man's problems have been successfully solved by a combination of intuitive talent and a desire to accept the challenge presented by a new problem. There is a place for both the analytical and the intuitive methods of attack. Neither can succeed completely without the other. Both require creative ability. It is difficult to write about general creativity, but since the approach to it used with the class was successful, it is included here in Chapter 12 as an essay.

We wish to warn the reader at the outset that even though we may attempt partly to inform and partly to stimulate him, however short we may fall of this goal, we have little intention of settling here questions which should not and cannot be settled in the few pages of one book. These remarks apply in the same measure to the mathematical methods given to be used primarily in increasing understanding of problems and secondarily in giving solutions to these problems.

Many friends have contributed to the preparation of this book by suggestions, criticism, and proofreading. My indebtedness to various authors, journals, and books is clear in the references. Here, my thanks to Dr. George Shortley, editor of *Operations Research*, for useful suggestions and permission to use my paper from the journal as Chapter 11. My gratitude for encouragement and guidance in the general structure of the book goes to my Operations Evaluation Group colleague Dr. Robert

S. Titchen and to my friend Dr. George G. O'Brien. In occasional discussions, many useful ideas and examples and criticisms came from my colleagues Dr. Richard H. Brown, Dr. John Danskin, and Robert P. Smith.

In particular, I wish to thank Dr. Allan J. Goldman, of the National Bureau of Standards, for his careful reading of and valuable suggestions on the optimization section, and Dr. Joan Rosenblatt for illuminating discussions on probability and statistics. To my tireless friend Theodore Wehe go my thanks for proofreading and frequent help in assembling and writing the first and second chapters, and to Monte Bourjaily, for encouragement and editorial help on the essay on creativity. To my Yale professor Carl Hempel (now at Princeton) go my thanks for lending me ideas and material from a lecture on symbolic logic.

I am indebted to Professor Sir Ronald A. Fisher, Cambridge, to Dr. Frank Yates, Rothamsted, and to Oliver & Boyd, Ltd., Edinburgh and London, for permission to reprint Tables 10-5 to 10-10 from their book "Statistical Tables for Biological, Agricultural, and Medical Research."

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